

A WORMHOLE-GENERATED PHYSICAL UNIVERSE WITH REFINED APPROXIMATION*

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Abstract

We improved the approximation of the model of the wormhole generated physical universe constructed by Choudhury and Pendharkar. We show here that the negative pressure of the unphysical wormhole can generate the right physical condition to explain the correct sign of the deacceleration parameter q_0 . The negative sign of q_0 implies an accelerating universe.

1 Introduction

The model constructed by Choudhury and Pendharkar [2](see also [1]) is one where our physical universe is subjected to a negative pressure originating from the wormhole occupying the core of the physical universe. It was assumed that the physical universe is separated from the wormhole by a thin wall which keeps the contents of these two entities separate. However the pressure generated in the wormhole is assumed to satisfy the adiabatic gas law. The wormhole core is a modified Gidding-Strominger [3],[4] wormhole. The scale factor of the wormhole has been calculated in an Euclidean space. We then introduce the real time by analytic continuation assuming that such continuation is mathematically permissible. The scale factor of such a wormhole is found to have both positive and negative signs. We chose the negative sign for the wormhole. This is of an impenetrable and unphysical wormhole core. It is one possible way to explain some physical properties of the real world. Such mathematical devices to explain physical properties of the real world have been used before for example by the use of indefinite metric in field theories. We consider here that the wormhole core is an unphysical interior. However it can generate relevant effects in the physical universe. We find that the negative pressure is such an effect. The generation of such pressure can easily explain the accelerating universe.

In our previous paper we have reached the conclusion that we live in an accelerating universe under certain approximation. In this paper we improve our approximation further and find our result does not change.

In section 2 we give a short introduction of the basics of the modified Gidding-Strominger wormhole model for the sake of completeness. In section 3, we show, following Choudhury and Pendharkar, how to calculate pressure. This pressure is generated inside the wormhole core and transferred to

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the physical universe. We also state the relevance of the approximation. In section 4 we derive the ratio $\frac{dR}{dt}/R$ of our physical universe. In section 5 we calculated the time dependent H_0 and the deacceleration parameter q_0 . The discussion of results follows in section 6.

2 The wormhole core

Choudhury and Pendharkar [2] have developed a model where they have generated a negative pressure from a wormhole located at the core of our physical universe. The core wormhole is a modified Gidding-Strominger one. Choudhury and Pendharkar have shown that one of the outcomes of the model is a negative pressure under certain approximation. In this paper we intend to relax the restriction further and show that the model's main outcome still holds.

As usual we start from a modified Gidding-Strominger wormhole action

$$S_E = \int d^4x L_G^c(x) + \int d^4x L_{SA}^c(x) = S_G + S_A, \quad (1)$$

where

$$L_G^c(x) = \frac{\sqrt{g^c} R(g^c)}{2\kappa^2}, \quad (2)$$

and

$$L_{SA}^c(x) = \sqrt{g^c} \left[\frac{1}{2} (\nabla \Phi)^2 + g_p^2 \Phi^2 \text{Exp}(\beta \Phi^2) \right] H_{\mu\nu\rho}^2. \quad (3)$$

In the above expression as well as in the subsequent formulae the suffix c stands for the core. Following Gidding and Strominger, we introduce

$$H_{\mu\nu\rho} = \frac{n}{g_p^2 a^3(t)} \varepsilon_{\mu\nu\rho}, \quad (4)$$

We get Eq.(3)

$$H^2 = \frac{6D}{g_p^2 a^6}, \quad (5)$$

with

$$D = \frac{n^2}{g_p^2} \quad (6)$$

The space-time interval in Euclidean space is given by

$$ds^2 = dt^2 + a^2(t)(d\chi^2 + \sin^2\chi d\theta^2 + \sin^2\chi \sin^2\phi d\phi^2). \quad (7)$$

The variation of $g_{\mu\nu}^c$ in the core leads to the following equation

$$G_{\mu\nu}^c = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu}^c R^c = \kappa^2 \left[\nabla_\mu \Phi \nabla_\nu \Phi - \left(\frac{1}{2} (\nabla \Phi)^2 \right) g_{\mu\nu}^c + g_p^2 \text{Exp}(\beta \Phi^2) (H_\mu{}^\alpha{}_\gamma H_\nu{}^\alpha{}_\gamma - \frac{1}{6} g_{\mu\nu}^c H_{\alpha\beta\gamma}^2) \right] \quad (8)$$

Using the Φ -variation inside the core we find the equation of motion of Φ

$$\nabla^2 \Phi - 2g_p^2 \beta \Phi \text{Exp}(\beta \Phi^2) H_{\alpha\beta\rho}^2 = 0. \quad (9)$$

The Hamiltonian constraint yields

$$\left(\frac{1}{a} \frac{da}{dt}\right)^2 - \frac{1}{a^2} = \frac{\kappa^2}{3} \left[\frac{1}{2} \left(\frac{d\Phi}{dt}\right)^2 - 6 \text{Exp}(\beta\Phi^2) \frac{n^2}{g_p^2 a^6} \right]. \quad (10)$$

The dynamical equation yields

$$\frac{d}{dt} \left(\frac{1}{a} \frac{da}{dt} \right) + \frac{1}{a^2} = -\kappa^2 \left[\left(\frac{d\Phi}{dt}\right)^2 - 12 \text{Exp}(\beta\Phi^2) \frac{n^2}{g_p^2 a^6} \right]. \quad (11)$$

Introducing a new variable τ defined by the relation

$$d\tau = a^{-3} dt, \quad (12)$$

the Eq.(9) reduces to

$$\frac{d^2\Phi}{d\tau^2} - 2D\beta \text{Exp}(\beta\Phi^2) = 0. \quad (13)$$

The Eqs.(9)- (11) yield (see [2] for details)

$$\left(\frac{1}{a} \frac{da}{dt}\right)^2 - a^4 + a_c^4 = 0, \quad (14)$$

where

$$a_c = \sqrt{\left(\frac{\kappa^2 C_o}{2}\right)} \quad (15)$$

and C_o is a constant which appears while converting Eq.(13) into an integral equation [2]. The solution of the Eq.(14) has been obtained by Gidding and Strominger and is given by

$$a^2(\tau) = a_c^2 \sqrt{\sec(2a_c^2 \tau)} \quad (16)$$

The scale factor can now be of two possible forms

$$a(\tau) = \pm a_c (\sec(2a_c^2 \tau))^{\frac{1}{4}}, \quad (17)$$

specified by the signs. Both the solutions stand on equal footing.

3 Pressure generated by the wormhole

Following Choudhury and Pendharkar [2] we assume that the wormhole is in a gaseous state satisfying the adiabatic gas law

$$P_w V_w^\gamma = \text{Constant} = B_1, \quad (18)$$

where γ is a constant. The volume of the wormhole can be shown to be proportional to $a^3(\tau)$. The pressure can be expressed as

$$P_w = (\pm 1)^{-3\gamma} B a_c^{-3\gamma} [\sec(2a_c^2 \tau)]^{-(3\gamma/4)} \quad (19)$$

where B is a new constant. We now switch from Euclidean space to Lorentz space. Instead of the zeroeth approximation [2], we replaced, in Eq.(12), a with a_c , and t with (it) . In our present approximation we get

$$d\tau = \pm a_c^{-3} \frac{dt}{[\cosh \frac{2t}{a_c}]^{3/4}} \quad (20)$$

For large t we neglect the negative exponential in cosh term and obtain for τ the value

$$\tau = -(\pm) \frac{1}{3a_c^2} \text{Exp}(-\frac{3t}{2a_c}) \quad (21)$$

Substituting this value of τ Eq.(17) $a(\tau)$ changes into

$$a(\tau) = \pm a_c [\sec(\frac{1}{3} \text{Exp}(-\frac{3t}{2a_c}))]^{1/4} \quad (22)$$

Following Choudhury and Pendharkar [2] we can convert Eq.(19) into

$$P_w = \pm B a_c^{-3\gamma} [\cos(\frac{2}{3} \text{Exp}(-\frac{3t}{2a_c}))]^{(3\gamma/4)}. \quad (23)$$

As usual we have restricted γ by the equation

$$3\gamma = 4n + 1 \quad (24)$$

where n is an integer. We choose in our model only the negative sign for the pressure P_w , i.e.,

$$P_w = -B a_c^{-3\gamma} (\cos[\frac{2}{3} \text{Exp}(-\frac{3t}{2a_c})])^{(3\gamma/4)} \quad (25)$$

4 The Robertson-Walker space with a wormhole at its core

As assumed by Choudhury and Pendharkar we live in a physical universe with a wormhole at the core. We reiterate the assumption that there is a wall which separates the physical universe from the content of the wormhole but the pressure generated in the wormhole can be transferred to the physical universe through the wall. Thus the Robertson-Walker metric is given by the relation

$$ds_R^2 = -dt^2 + R^2(t) [\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2] \quad (26)$$

where we have chosen $c=1$. The Einstein equation is as follows

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = -\frac{8\pi G}{3} S_{\mu\nu} = -\frac{8\pi G}{3} (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^\lambda{}_\lambda). \quad (27)$$

We assume that the energy momentum tensor has the perfect fluid form

$$T_{\mu\nu} = P_T g_{\mu\nu} + (P_T + \rho) U_\mu U_\nu \quad (28)$$

Following Choudhury and Pendharkar we can straight forward deduce

$$(\frac{dR}{dt})^2 + k = -\frac{8\pi G}{3} P_T R^2. \quad (29)$$

In this paper we only study the case of $k=0$. We find that

$$\frac{(\frac{dR(t)}{dt})^2}{R^2(t)} = -\frac{8\pi G}{3}P_T. \quad (30)$$

Assuming further that the fluid pressure of the physical world is significantly smaller than the wormhole pressure, we can replace P_T by P_W of Eq.(25). Therefore we can write

$$\frac{(\frac{dR(t)}{dt})}{R(t)} = \pm\sqrt{(-\frac{8\pi G}{3}P_W)}. \quad (31)$$

where P_W is given by the Eq.(25)

5 Hubble and deacceleration parameters

Following Choudhury and Pendharkar we can show that

$$\frac{\frac{dR(t)}{dt}}{R(t)} = \pm i^{-3\gamma+1+4n} \sqrt{(\frac{8\pi GB}{3})a_c^{-3\gamma/2}(\cos[\frac{2}{3}\text{Exp}[-\frac{2t}{a_c}]])^{3\gamma/8}}. \quad (32)$$

We choose n to be a positive integer and to satisfy the condition

$$3\gamma = 4n + 1. \quad (33)$$

Choosing now the positive sign in Eq.(32)we get

$$\frac{\frac{dR(t)}{dt}}{R(t)} = \sqrt{(\frac{8\pi GB}{3})a_c^{-3\gamma/2}(\cos[\frac{2}{3}\text{Exp}[-\frac{2t}{a_c}]])^{3\gamma/8}}. \quad (34)$$

For $t = t_o$, we get the time dependent Hubble parameter

$$H_o(t_o) = \sqrt{(\frac{8\pi GB}{3})a_c^{-3\gamma/2}(\cos[\frac{2}{3}\text{Exp}[-\frac{2t_o}{a_c}]])^{3\gamma/8}}. \quad (35)$$

For large t_o we get

$$H_o(\infty) = \sqrt{(\frac{8\pi GB}{3})a_c^{-3\gamma/2}} \quad (36)$$

We can now easily find the deacceleration parameter. Following Choudhury and Pendharkar we can show that q_o turns out to be:

$$q_o(t_o) = 4\pi GB[\gamma a_c^{-(3\gamma+1)}e^{-\frac{2t_o}{a_c}}\cos^{\frac{3\gamma-4}{4}}(\frac{2}{3}e^{-\frac{2t_o}{a_c}})\sin(\frac{2}{3}e^{-\frac{2t_o}{a_c}})H_o^{-3} - 2a_c^{-3\gamma}\cos^{\frac{3\gamma}{4}}(\frac{2}{3}e^{-\frac{2t_o}{a_c}})H_o^{-2}]. \quad (37)$$

For large t_o , $q_o(t_o)$ becomes

$$q_o(t_o) = -\frac{8\pi GB}{3}a_c^{-3\gamma}\cos^{\frac{3\gamma}{4}}(\frac{2}{3}e^{-\frac{2t_o}{a_c}})H_o^{-2}. \quad (38)$$

This expression is always negative. As a result the universe expands at an accelerating rate for large t_o . For infinite t_o the deacceleration parameter becomes

$$q_o(\infty) = -1. \quad (39)$$

6 Concluding Remarks

Using the model elaborated by Choudhury and Pendharkar and improving the approximation more rigorously, we have again reached the conclusion that the physical universe accelerates for large value of the present time t_o . In [2] the approximation τ connects to the real time t by the equation

$$\tau = ia_c^3 t. \quad (40)$$

Whereas in our present approximation we get Eq.[21]

$$\tau = \pm \frac{1}{3a_c^2} e^{-\frac{3t}{2a_c}}. \quad (41)$$

With our new approximation we find that the physical universe driven by adiabatic pressure expands and accelerates. This is one way to explain the present experimental data [5] without introducing dark energies [6] in the picture.

It would be really interesting to estimate the parameters introduced in the theory by comparing them with the experimental results. Work is in progress along this line.

7 References

1. A. L. Choudhury, Hadronic J., 23, 581 (2000).
2. L. Choudhury and H. Pendharkar, Hadronic J. 24, 275 (2001).
3. S. B. Giddings and A. Strominger , Nucl. Phys. B 307, 854 (1988).
4. D. H. Coule and K. Maeda, Class. Quant. Grav. 7, 955 (1990).
5. N. Bahcall, J. P. Ostriker, S. Perlmutter, and P. J. Steinhardt, Science, 284,1481 (1999).
6. C. Arendariz Picon, V. Mukhanov and Paul J. Steinhardt: Essentials of k-Essence. ArXiv:astro-ph/0006373 (2000).